Probability:

AMC 12 1999

24. Six points on a circle are given. Four of the chords joining pairs of the six points are selected at random. What is the probability that the four chords form a convex quadrilateral?

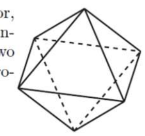
(A) $\frac{1}{15}$ **(B)** $\frac{1}{91}$ **(C)** $\frac{1}{273}$ **(D)** $\frac{1}{455}$ **(E)** $\frac{1}{1365}$

AMC 12 2000

23. Professor Gamble buys a lottery ticket, which requires that he pick six different integers from 1 through 46, inclusive. He chooses his numbers so that the sum of the base-ten logarithms of his six numbers is an integer. It so happens that the integers on the winning ticket have the same property– the sum of the base-ten logarithms is an integer. What is the probability that Professor Gamble holds the winner ticket?

(A) 1/5 **(B)** 1/4 **(C)** 1/3 **(D)** 1/2 **(E)** 1

25. Eight congruent equilateral triangles, each of a different color, are used to construct a regular octahedron. How many distinguishable ways are there to construct the octahedron? (Two colored octahedrons are distinguishable if neither can be rotated to look just like the other.)



(A) 210 **(B)** 560 **(C)** 840

(D) 1260 **(E)** 1680

Amc 12 2001

11. A box contains exactly five chips, three red and two white. Chips are randomly removed one at a time without replacement until all the red chips are drawn or all the white chips are drawn. What is the probability that the last chip drawn is white?

(A) $\frac{3}{10}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{7}{10}$

14. Given the nine-sided regular polygon $A_1A_2A_3A_4A_5A_6A_7A_8A_9$, how many distinct equilateral triangles in the plane of the polygon have at least two vertices in the set $\{A_1, A_2, \ldots A_9\}$?

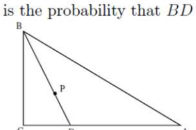
(A) 30 **(B)** 36 **(C)** 63 **(D)** 66 **(E)** 72

16. A spider has one sock and one shoe for each of its eight legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

(A) $8!$ **(B)** $2^8 8!$ **(C)** $(8!)^2$ **(D)** $\frac{16!}{2^8}$ **(E)** $16!$

AMC 12 2002a

22. Triangle ABC is a right triangle with $\angle ACB$ as its right angle, $m\angle ABC = 60°$, and $AB = 10$. Let P be randomly chosen inside $\triangle ABC$, and extend $\overline{BP}$ to meet $\overline{AC}$ at D. What is the probability that $BD > 5\sqrt{2}$?



(A) $\frac{2-\sqrt{2}}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3-\sqrt{3}}{3}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{5-\sqrt{5}}{5}$

16. Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$, and Sergio randomly selects a number from the set $\{1, 2, \ldots, 10\}$. The probability that Sergio's number is larger than the sum of the two numbers chosen by Tina is

(A) 2/5 **(B)** 9/20 **(C)** 1/2 **(D)** 11/20 **(E)** 24/25

AMC 12 2002b

16. Juan rolls a fair regular octahedral die marked with the numbers 1 through 8. Then Amal rolls a fair six-sided die. What is the probability that the product of the two rolls is a multiple of 3?

(A) $\frac{1}{12}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{7}{12}$ **(E)** $\frac{2}{3}$

18. A point P is randomly selected from the rectangular region with vertices $(0,0), (2,0)$, $(2,1), (0,1)$. What is the probability that P is closer to the origin than it is to the point $(3,1)$?

(A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{4}{5}$ **(E)** 1

AMC 12 2003a

8. What is the probability that a randomly drawn positive factor of 60 is less than 7?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

20. How many 15-letter arrangements of 5 A's, 5 B's, and 5 C's have no A's in the first 5 letters, no B's in the next 5 letters, and no C's in the last 5 letters?

(A) $\sum_{k=0}^{5} \binom{5}{k}^3$ **(B)** $3^5 \cdot 2^5$ **(C)** 2^{15} **(D)** $\frac{15!}{(5!)^3}$ **(E)** 3^{15}

Amc 12 2003b

19. Let S be the set of permutations of the sequence $1, 2, 3, 4, 5$ for which the first term is not 1. A permutation is chosen randomly from S. The probability that the second term is 2, in lowest terms, is a/b. What is $a + b$?

(A) 5 (B) 6 (C) 11 (D) 16 (E) 19

25. Three points are chosen randomly and independently on a circle. What is the probability that all three pairwise distances between the points are less than the radius of the circle?

(A) $\frac{1}{36}$ (B) $\frac{1}{24}$ (C) $\frac{1}{18}$ (D) $\frac{1}{12}$ (E) $\frac{1}{9}$

Amc 12 2004b

20. Each face of a cube is painted either red or blue, each with probability 1/2. The color of each face is determined independently. What is the probability that the painted cube can be placed on a horizontal surface so that the four vertical faces are all the same color?

(A) $\frac{1}{4}$ (B) $\frac{5}{16}$ (C) $\frac{3}{8}$ (D) $\frac{7}{16}$ (E) $\frac{1}{2}$

Amc 12 2005a – dependent events

14. On a standard die one of the dots is removed at random with each dot equally likely to be chosen. The die is then rolled. What is the probability that the top face has an odd number of dots?

(A) $\frac{5}{11}$ (B) $\frac{10}{21}$ (C) $\frac{1}{2}$ (D) $\frac{11}{21}$ (E) $\frac{6}{11}$

Amc 12 2005b

11. An envelope contains eight bills: 2 ones, 2 fives, 2 tens, and 2 twenties. Two bills are drawn at random without replacement. What is the probability that their sum is \$20 or more?

(A) $\frac{1}{4}$ (B) $\frac{2}{7}$ (C) $\frac{3}{7}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

25. Six ants simultaneously stand on the six vertices of a regular octahedron, with each ant at a different vertex. Simultaneously and independently, each ant moves from its vertex to one of the four adjacent vertices, each with equal probability. What is the probability that no two ants arrive at the same vertex?

(A) $\frac{5}{256}$ (B) $\frac{21}{1024}$ (C) $\frac{11}{512}$ (D) $\frac{23}{1024}$ (E) $\frac{3}{128}$

AMC 12 2006a

20. A bug starts at one vertex of a cube and moves along the edges of the cube according to the following rule. At each vertex the bug will choose to travel along one of the three edges emanating from that vertex. Each edge has equal probability of being chosen, and all choices are independent. What is the probability that after seven moves the bug will have visited every vertex exactly once?

(A) $\frac{1}{2187}$ (B) $\frac{1}{729}$ (C) $\frac{2}{243}$ (D) $\frac{1}{81}$ (E) $\frac{5}{243}$

AMC 12 2006b - dependent events

17. For a particular peculiar pair of dice, the probabilities of rolling 1, 2, 3, 4, 5, and 6 on each die are in the ratio $1 : 2 : 3 : 4 : 5 : 6$. What is the probability of rolling a total of 7 on the two dice?

(A) $\frac{4}{63}$ (B) $\frac{1}{8}$ (C) $\frac{8}{63}$ (D) $\frac{1}{6}$ (E) $\frac{2}{7}$

AMC 12 2007a

12. Integers a, b, c, and d, not necessarily distinct, are chosen independently and at random from 0 to 2007, inclusive. What is the probability that $ad - bc$ is even?

(A) $\frac{3}{8}$ (B) $\frac{7}{16}$ (C) $\frac{1}{2}$ (D) $\frac{9}{16}$ (E) $\frac{5}{8}$

AMC 12 2007b

13. A traffic light runs repeatedly through the following cycle: green for 30 seconds, then yellow for 3 seconds, and then red for 30 seconds. Leah picks a random three-second time interval to watch the light. What is the probability that the color changes while she is watching?

(A) $\frac{1}{63}$ (B) $\frac{1}{21}$ (C) $\frac{1}{10}$ (D) $\frac{1}{7}$ (E) $\frac{1}{3}$

AMC 12 2008a

21. A permutation $(a_1, a_2, a_3, a_4, a_5)$ of $(1, 2, 3, 4, 5)$ is heavy-tailed if $a_1 + a_2 < a_4 + a_5$. What is the number of heavy-tailed permutations?

(A) 36 (B) 40 (C) 44 (D) 48 (E) 52

AMC 12 2008b

22. A parking lot has 16 spaces in a row. Twelve cars arrive, each of which requires one parking space, and their drivers choose their spaces at random from among the available spaces. Auntie Em then arrives in her SUV, which requires 2 adjacent spaces. What is the probability that she is able to park?

(A) $\frac{11}{20}$ (B) $\frac{4}{7}$ (C) $\frac{81}{140}$ (D) $\frac{3}{5}$ (E) $\frac{17}{28}$

AMC 12 2009b

Problem 21

Ten women sit in 10 seats in a line. All of the 10 get up and then reseat themselves using all 10 seats, each sitting in the seat she was in before or a seat next to the one she occupied before. In how many ways can the women be reseated?

(A) 89 (B) 90 (C) 120 (D) 2^{10} (E) $2^2 3^8$

Problem 18

Rachel and Robert run on a circular track. Rachel runs counterclockwise and completes a lap every 90 seconds, and Robert runs clockwise and completes a lap every 80 seconds. Both start from the start line at the same time. At some random time between 10 minutes and 11 minutes after they begin to run, a photographer standing inside the track takes a picture that shows one-fourth of the track, centered on the starting line. What is the probability that both Rachel and Robert are in the picture?

(A) $\frac{1}{16}$ (B) $\frac{1}{8}$ (C) $\frac{3}{16}$ (D) $\frac{1}{4}$ (E) $\frac{5}{16}$

Amc 12 2010a

Problem 15

A coin is altered so that the probability that it lands on heads is less than $\frac{1}{2}$ and when the coin is flipped four times, the probability of an equal number of heads and tails is $\frac{1}{6}$. What is the probability that the coin lands on heads?

(A) $\frac{\sqrt{15}-3}{6}$ (B) $\frac{6-\sqrt{6\sqrt{6}+2}}{12}$ (C) $\frac{\sqrt{2}-1}{2}$ (D) $\frac{3-\sqrt{3}}{6}$ (E) $\frac{\sqrt{3}-1}{2}$

Problem 16

Bernardo randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

(A) $\frac{47}{72}$ (B) $\frac{37}{56}$ (C) $\frac{2}{3}$ (D) $\frac{49}{72}$ (E) $\frac{39}{56}$

Problem 19

Each of 2010 boxes in a line contains a single red marble, and for $1 \le k \le 2010$, the box in the kth position also contains k white marbles. Isabella begins at the first box and successively draws a single marble at random from each box, in order. She stops when she first draws a red marble. Let $P(n)$ be the probability that Isabella stops after drawing exactly n marbles. What is the smallest value of n for which $P(n) < \frac{1}{2010}$?

(A) 45 (B) 63 (C) 64 (D) 201 (E) 1005

AMC 12 2010b

11. A palindrome between 1000 and 10,000 is chosen at random. What is the probability that it is divisible by 7?

(A) $\frac{1}{10}$ (B) $\frac{1}{9}$ (C) $\frac{1}{7}$ (D) $\frac{1}{6}$ (E) $\frac{1}{5}$

18. A frog makes 3 jumps, each exactly 1 meter long. The directions of the jumps are chosen independently and at random. What is the probability that the frog's final position is no more than 1 meter from its starting position?

(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

16. Positive integers a, b, and c are randomly and independently selected with replacement from the set $\{1, 2, 3, \ldots, 2010\}$. What is the probability that $abc + ab + a$ is divisible by 3?

(A) $\frac{1}{3}$ (B) $\frac{29}{81}$ (C) $\frac{31}{81}$ (D) $\frac{11}{27}$ (E) $\frac{13}{27}$

AMC 12 2011a

10. A pair of standard 6-sided fair dice is rolled once. The sum of the numbers rolled determines the diameter of a circle. What is the probability that the numerical value of the area of the circle is less than the numerical value of the circle's circumference?

(A) $\frac{1}{36}$ (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) $\frac{1}{4}$ (E) $\frac{5}{18}$

Euclid 2016

9.  (a) The string $AAABBBAABB$ is a string of ten letters, each of which is A or B, that does not include the consecutive letters $ABBA$.
The string $AAABBAAABB$ is a string of ten letters, each of which is A or B, that does include the consecutive letters $ABBA$.
Determine, with justification, the total number of strings of ten letters, each of which is A or B, that do not include the consecutive letters $ABBA$.

Euclid 2014 dependent events

(a) A bag contains 40 balls, each of which is black or gold. Feridun reaches into the bag and randomly removes two balls. Each ball in the bag is equally likely to be removed. If the probability that two gold balls are removed is $\frac{5}{12}$, how many of the 40 balls are gold?

AIME Problems on Counting and Probability

AIME II) 2015

Problem

The nine delegates to the Economic Cooperation Conference include 2 officials from Mexico, 3 officials from Canada, and 4 officials from the United States. During the opening session, three of the delegates fall asleep. Assuming that the three sleepers were determined randomly, the probability that exactly two of the sleepers are from the same country is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

The total number of ways to pick 3 officials from 9 total is $\binom{9}{3} = 84$, which will be our denominator. Now we can consider the number of ways for exactly two sleepers to be from the same country for each country individually and add them to find our numerator:

- There are 7 different ways to pick 3 delegates such that 2 are from Mexico, simply because there are $9-2=7$ "extra" delegates to choose to be the third sleeper once both from Mexico are sleeping.
- There are $3 \times 6 = 18$ ways to pick from Canada, as each Canadian can be left out once and each time one is left out there are $9-3=6$ "extra" people to select one more sleeper from.
- Lastly, there are $6 \times 5 = 30$ ways to choose for the United States. It is easy to count 6 different ways to pick 2 of the 4 Americans, and each time you do there are $9-4=5$ officials left over to choose from.

Thus, the fraction is $\frac{7+18+30}{84} = \frac{55}{84}$. Since this does not reduce, the answer is $55+84 = \boxed{139}$.

Solution 2

The total number of ways to pick 3 officials from 9 total is 84. We note that two sleepers are asleep and one is awake if and only if the sleepers come from two distinct countries. The sleeping officials can be from either 1, 2, or 3 countries.

- If the sleeping officials are from a single country, they can be from Canada in 1 way and from the United States in 4 ways, so there are 5 total possibilities.
- If the sleeping officials are from 3 different countries, there must be one from each. So there are $2 \cdot 3 \cdot 4 = 24$ total possibilities.

Out of 84 total, $5+24=29$ possibilities are different from the case we are looking for, so there are $84-29=55$ total ways to choose 22 officials from one country and a single official from another country. As 55 and 84 share no common factors, we have $55+84=\boxed{139}$.

2015 AIME I Problems/Problem 5

In a drawer Sandy has 5 pairs of socks, each pair a different color. On Monday Sandy selects two individual socks at random from the 10 socks in the drawer. On Tuesday Sandy selects 2 of the remaining 8 socks at random and on Wednesday two of the remaining 6 socks at random. The probability that Wednesday is the first day Sandy selects matching socks is $\frac{m}{n}$, where m and n are relatively prime positive integers, Find $m+n$.

Solution 1

Let the fifth sock be arbitrary; the probability that the sixth sock matches in color is $\frac{1}{9}$.

Assuming this, then let the first sock be arbitrary; the probability that the second sock does not match is $\frac{6}{7}$.

The only "hard" part is the third and fourth sock. But that is simple casework. If the third sock's color matches the color of one of the first two socks (which occurs with probability $\frac{2}{6} = \frac{1}{3}$), then the fourth sock can be arbitrary. Otherwise (with probability $\frac{2}{3}$), the fourth sock can be chosen with probability $\frac{4}{5}$ (5 socks left, 1 sock that can possibly match the third sock's color). The desired probability is thus

$$\frac{1}{9} \cdot \frac{6}{7} \cdot (\frac{1}{3} + \frac{2}{3} \cdot \frac{4}{5}) = \frac{26}{315}.$$

Solution 2

The key is to count backwards. First, choose the pair which you pick on Wednesday in 5 ways. Then there are four pairs of socks for you to pick a pair of on Tuesday, and you don't want to pick a pair. Since there are 4 pairs, the number of ways to do this is $\binom{8}{2} - 4$. Then, there are two pairs and two nonmatching socks for you to pick from on Monday, a total of 6 socks. Since you don't want to pick a pair, the number of ways to do this is $\binom{6}{2} - 2$. Thus the answer is

$$\frac{(5)\left(\binom{8}{2} - 4\right)\left(\binom{6}{2} - 2\right)}{\binom{10}{2}\binom{8}{2}\binom{6}{2}} = \frac{26}{315}.$$

$26 + 315 = \boxed{341}$.